J. Fluid Mech. (2002), *vol.* 459, *pp.* 129–138.
DOI: 10.1017/S0022112002008248 Printed in the United Kingdom

Random-sweeping hypothesis for passive scalars in isotropic turbulence

By P. K. YEUNG[1] AND BRIAN L. SAWFORD[2]

[1]School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

[2]CSIRO Atmospheric Research, Private Bag No. 1, Aspendale, VIC 3195, Australia

(Received 20 December 2001 and in revised form 10 February 2002)

The hypothesis of the small scales being passively swept along by the large-scale motions in turbulent flow is extended to passive scalars in isotropic turbulence. A theory based on strong mutual cancellation between local and advective derivatives and other assumptions is shown to capture the Reynolds and Schmidt number dependence of time scales characterizing Eulerian and Lagrangian rates of change. Agreement with direct numerical simulation data improves systematically with increasing Reynolds number. In accordance with the physics of random sweeping, the Eulerian frequency spectrum is very similar in shape to the wavenumber spectrum, but is broadened at higher frequencies compared to its Lagrangian counterpart. Overall the hypothesis appears to be even more valid for transported scalars than for the velocity field, which gives support to the use of Lagrangian approaches in the study of turbulent mixing.

1. Introduction

An important concept in the study of turbulent transport at high Reynolds number is the assumption that the small scales are passively swept past a fixed observer by the large eddies in the flow. The physics of this 'random sweeping' implies that changes in the Lagrangian frame following the path of fluid particle motion dominated by the large scales are small. The dominance of advective transport by the velocity fluctuations is a primary reason behind the common use and success of Lagrangian approaches in the study of turbulent transport processes, including the modelling of contaminant dispersion in the environment (see e.g. a recent review by Sawford 2001). In addition, in analytical theories of turbulence a proper treatment of the effects of random sweeping is critical in the use of Lagrangian methods to derive two-point models of Eulerian inertial-range scaling behaviour in turbulence and passive scalar fields. This particular aspect has been demonstrated in classical work by Kraichnan (1966), as well as more recent use of Lagrangian or quasi-Lagrangian variables (Belinicher & L'vov 1987) to investigate scaling exponents in the structure functions of passive scalars (Fairhall *et al.* 1996). Similar arguments based on sweeping by the mean velocity are also central to the Taylor frozen-turbulence hypothesis, which is often invoked in experiments to infer space derivatives from temporal surrogates recorded by a fixed sensor in space.

Our primary motivation in this work is to use random sweeping concepts to provide further information on Lagrangian quantities important in the modelling of mixing and dispersion. Of particular interest in this context is the work of Tennekes (1975), who used these ideas to develop theoretical estimates of Eulerian and Lagrangian time microscales representing time derivatives in the corresponding reference frames, and to compare the shapes of Eulerian and Lagrangian spectra in the frequency domain. Although Tennekes' results were, on a quantitative level, affected by the use of data at low Reynolds number, qualitatively his hypothesis appears to be supported well by many later investigations (e.g. direct numerical simulations (DNS) in Yeung & Pope 1989 and Tsinober, Vedula & Yeung 2001).

We consider here, in a framework somewhat similar to Tennekes (1975), the extension of random-sweeping concepts to passive scalar quantities transported in turbulent flow. Recent results from DNS (Yeung 2001) for passive scalar fluctuations maintained by a uniform mean gradient in forced stationary isotropic turbulence have shown that a marked contrast exists between the evolution of scalar fluctuations in Eulerian versus Lagrangian frames. In particular, especially at higher Reynolds number and/or Schmidt number, scalar fluctuations were found to evolve more quickly than the velocity in the Eulerian frame, but less so in the Lagrangian frame. One of our objectives below is to perform an analysis of the Eulerian and Lagrangian rates of change, in a way that gives direct information on how these quantities depend on Reynolds number and Schmidt number. We obtain several theoretical estimates and compare them with results from the analysis of DNS databases presented in Yeung (2001), and from a new series of simulations at low Reynolds number but with a fairly wide range of Schmidt numbers from 1/4 to 64. A second objective is to explore the functional forms of Eulerian and Lagrangian spectra of the scalars in relation to each other. These two objectives are addressed respectively in §§ 2 and 3. Conclusions are summarized in § 4.

2. Time scale estimates

2.1. *Eulerian time scale*

We consider the fluctuations of a scalar of molecular diffusivity D governed by the transport equation in tensor form

$$\frac{\mathrm{D}\phi}{\mathrm{D}t} = \frac{\partial \phi}{\partial t} + u_i \frac{\partial \phi}{\partial x_i} = -u_i \frac{\partial \Phi}{\partial x_i} + D \frac{\partial^2 \phi}{\partial x_i \partial x_i}, \tag{2.1}$$

where $\mathrm{D}/\mathrm{D}t$ is the material derivative operator, and Φ is the mean scalar field with a uniform gradient. The material derivative $\mathrm{D}\phi/\mathrm{D}t$ is equivalent to the Lagrangian rate of change of ϕ, which was found in Yeung (2001) to have a much smaller variance than the Eulerian unsteady time derivative $\partial\phi/\partial t$. This suggests that, in a way similar to the work of Tennekes (1975, equation 1 therein) for the velocity field, we may assume a strong degree of cancellation between the unsteady (local) and advective (convective) contributions to the material derivative: that is,

$$\frac{\partial \phi}{\partial t} \approx -u_i \frac{\partial \phi}{\partial x_i}. \tag{2.2}$$

As in the case of the velocity field (Tsinober *et al.* 2001), the degree of mutual cancellation can be quantified by a correlation coefficient between the unsteady and advective terms. The hypothesis above implies that this correlation is close to −1.0: indeed for $Sc = 1.0$ we find it to be −0.956 at Taylor-scale Reynolds number (R_λ) 38 and −0.993 at R_λ 240.

Taking the variance of (2.2) gives the estimate

$$\left\langle \left(\frac{\partial \phi}{\partial t} \right)^2 \right\rangle = \left\langle u_i u_j \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_j} \right\rangle, \tag{2.3}$$

where angled brackets denote ensemble averaging. It is documented well elsewhere (Overholt & Pope 1996; Yeung 2001) that velocity fluctuations acting upon this mean gradient leads to a stationary state where the resulting production term for scalar variance is balanced by molecular dissipation. These properties of homogeneity and stationarity allow averaging to be taken in both space and time.

If we assume that the microstructure of the scalar field is statistically independent of the large-scale motions in the velocity field, and that perfect local isotropy applies, then the right-hand side of (2.3) can be rewritten in terms of the turbulence kinetic energy (K) and mean scalar dissipation rate ($\langle\chi\rangle \equiv 2D\langle(\partial\phi/\partial x_i)^2\rangle$). Accordingly, we obtain

$$\left\langle\left(\frac{\partial\phi}{\partial t}\right)^2\right\rangle \approx \frac{K\langle\chi\rangle}{3D}. \tag{2.4}$$

We define the Eulerian Taylor time scale to be that characterizing the Eulerian unsteady rate of change, as

$$\tau_{E,\phi} = \left[\frac{\langle\phi^2\rangle}{\langle(\partial\phi/\partial t)^2\rangle}\right]^{1/2}. \tag{2.5}$$

If we normalize this by the Kolmogorov time scale (τ_η) then use of (2.4) and (2.5) gives

$$\left(\frac{\tau_{E,\phi}}{\tau_\eta}\right)^2 = \frac{3}{Sc}\frac{1}{r_\phi}, \tag{2.6}$$

where $Sc \equiv \nu/D$ is the Schmidt number, and r_ϕ is the mechanical-to-scalar time scale ratio, defined by

$$r_\phi = \frac{K}{\langle\varepsilon\rangle}\bigg/\frac{\langle\phi^2\rangle}{\langle\chi\rangle}. \tag{2.7}$$

Figure 1 shows results for r_ϕ and $\tau_{E,\phi}/\tau_\eta$ obtained from DNS. The parameter ranges covered are $Sc = 1/8$ and 1 at R_λ 140 and 240 (Yeung 2001), and $Sc = 1/4$, 1/2, 1, 4, 8, 16, 32, and 64 at R_λ 38. (High Schmidt numbers are achieved in low Reynolds number simulations essentially by refined grid resolution, up to 512^3.) The data on r_ϕ show a mild but definite decrease with Sc, and a weaker dependence on Reynolds number especially at higher R_λ. In accordance with this, and as suggested by the estimate (2.6) itself, the Eulerian time-scale ratio $\tau_{E,\phi}/\tau_\eta$ is seen to be mainly determined by the Schmidt number. The fact that this ratio decreases strongly with Sc is consistent with observations from Eulerian time series (Yeung 2001) that, especially at higher Schmidt numbers, Eulerian scalar fluctuations evolve more rapidly than the velocity fluctuations.

It should be emphasized that high Reynolds number assumptions are used in the analysis leading to (2.4) above. Consequently, because of the Reynolds number limitations of DNS, perfect agreement is not to be expected. On the other hand, it is encouraging to observe that the best agreement between theory and DNS in figure 1 is that for the case of $Sc = 1$ at R_λ 240 (the highest R_λ in this work). The Schmidt number trend in DNS is also seen to be predicted well by the theory. In obtaining (2.4) from (2.3) we have neglected 'cross' terms of the type $\langle u_1u_2\partial\phi/\partial x_1\partial\phi/\partial x_2\rangle$, which are found to be negative. This causes (2.4) to give an overestimate of the time derivative compared to DNS, and correspondingly (2.6) to give an underestimate of the time-scale ratio $\tau_{E,\phi}/\tau_\eta$.

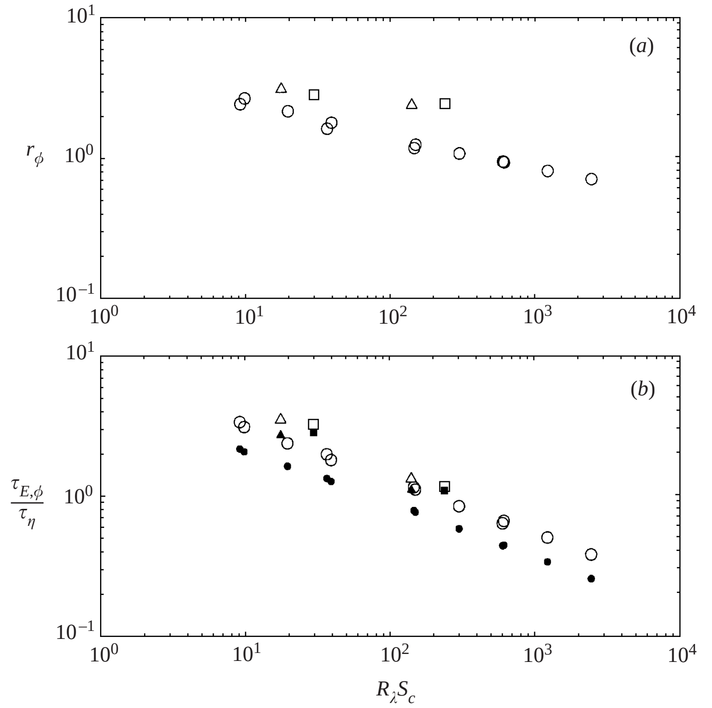

FIGURE 1. Variation of (*a*) mechanical-to-scalar time-scale ratio (r_ϕ) and (*b*) normalized Eulerian scalar Taylor time scale ($\tau_{E,\phi}/\tau_\eta$) as a function of the parameter $R_\lambda Sc$. Open symbols are from DNS data at different Reynolds numbers: ○, R_λ 38; △, R_λ 140; □, R_λ 240. Corresponding closed symbols in (*b*) are for estimates of $\tau_{E,\phi}/\tau_\eta$ according to (2.6).

2.2. *Lagrangian time scale*

To estimate the Lagrangian rate of change, we begin by noting that, in (2.1), the mean gradient ($-u_i\partial\Phi/\partial x_i$) and the molecular diffusion terms $D\nabla^2\phi$ are respectively dominated by the large scales and the small scales, and hence almost completely uncorrelated with each other (with the empirically determined correlation coefficient about 0.01 for $Sc = 1$ at R_λ 240). To determine the relative importance of these terms we can analyse their variances as follows. For a mean gradient of magnitude G aligned in the x_1-direction, the variance of $-u_i\partial\Phi/\partial x_i$ is simply $\langle u_1^2\rangle G^2$. Although this quantity is independent of the scalar fluctuations, it can be re-expressed in terms of scalar statistics by noting that the balance between production and dissipation of the scalar variance in the stationary state implies

$$-\langle u_1\phi\rangle G \approx \langle\chi\rangle \tag{2.8}$$

and hence

$$\langle u_1^2\rangle G^2 \approx \frac{\langle\chi\rangle^2}{\langle\phi^2\rangle(\rho_{u\phi})^2} \tag{2.9}$$

where $\rho_{u\phi}$ is the velocity–scalar cross-correlation coefficient.

For the variance of $D\nabla^2\phi$, Parseval's theorem in Fourier space implies that

$$\left\langle(D\nabla^2\phi)^2\right\rangle = D^2\int k^4 E_\phi(k)\,\mathrm{d}k, \tag{2.10}$$

where $E_\phi(k)$ is the scalar spectrum, and the integral is taken over all wavenumbers in the simulation. From Kerr's (1985) definition of the scalar dissipation skewness (S_χ, his equation 25), the variance of $D\nabla^2\phi$ can be written as

$$\langle (D\nabla^2\phi)^2 \rangle = S_\chi \frac{\sqrt{15}}{12} \frac{\langle \chi \rangle}{\tau_\eta}. \tag{2.11}$$

In our DNS data S_χ is nearly universal, at about 0.5. It may be noted that in the evolution equation for the scalar dissipation rate, stationarity implies the right-hand side of (2.10) is also proportional (by a factor equal to the molecular diffusivity) to a production term representing the nonlinear amplification of scalar gradients by fluctuating strain rates. Approximate universality for S_χ in (2.11) is thus consistent with the finding (Vedula, Yeung & Fox 2001) that nonlinear amplification in the budget of scalar dissipation rate scales with the Kolmogorov time scale.

Using the turbulence time-scale ratio $(K/\langle \varepsilon \rangle)/\tau_\eta \approx (3/2\sqrt{15})R_\lambda = 0.387R_\lambda$ together with (2.9) and (2.11) we can now write the ratio of molecular diffusion to mean gradient terms as

$$\frac{\langle (D\nabla^2\phi)^2 \rangle}{\langle u_1^2 \rangle G^2} \approx S_\chi \frac{\sqrt{15}}{12} (\rho_{u\phi})^2 \frac{R_\lambda}{r_\phi}. \tag{2.12}$$

With S_χ, $\rho_{u\phi}$ and r_ϕ all depending only weakly on Reynolds number, this estimate suggests that the molecular diffusion term will become large compared to the mean gradient term as R_λ is increased. (For $Sc = 1$, the ratio of variances increases from 3.3 at R_λ 140 to 6.8 at R_λ 240.) In other words, at high R_λ we expect

$$\left\langle (\mathrm{D}\phi/\mathrm{D}t)^2 \right\rangle \approx \left\langle (D\nabla^2\phi)^2 \right\rangle. \tag{2.13}$$

Comparison of (2.11) and (2.13) with (2.4) shows that the ratio of the variance of the Lagrangian rate of change to that of the Eulerian rate of change scales as $(R_\lambda Sc)^{-1}$, which allows us to quantify how quickly the terms neglected in (2.2) become weaker with increasing Reynolds and/or Schmidt numbers.

With the definition of the Lagrangian Taylor time scale as $\tau_{L,\phi} = [\langle \phi^2 \rangle / \langle (\mathrm{D}\phi/\mathrm{D}t)^2 \rangle]^{1/2}$, the above analysis leads to

$$\left(\frac{\tau_{L,\phi}}{\tau_\eta} \right)^2 \approx \frac{12}{\sqrt{15}} \frac{1}{S_\chi} \frac{\langle \phi^2 \rangle}{\langle \chi \rangle} \frac{1}{\tau_\eta}. \tag{2.14}$$

Using again the ratio $(K/\langle \varepsilon \rangle)/\tau_\eta \approx (3/2\sqrt{15})R_\lambda$ and setting $S_\chi = 0.5$, we finally obtain

$$\left(\frac{\tau_{L,\phi}}{\tau_\eta} \right)^2 \approx 2.4 \frac{R_\lambda}{r_\phi}. \tag{2.15}$$

The Lagrangian to Eulerian time-scale ratio from (2.6) and (2.15) is

$$\tau_{L,\phi}/\tau_{E,\phi} \approx \sqrt{0.8R_\lambda Sc}. \tag{2.16}$$

Comparisons for the ratios $\tau_{L,\phi}/\tau_\eta$ and $\tau_{L,\phi}/\tau_{E,\phi}$ are given in figure 2. As for figure 1, the agreement is best for data at the highest Reynolds number in the simulations, and the Schmidt number trend (via (2.15) and (2.16)) at a given R_λ is evidently captured very well. It can also be noted that, because of the neglect of the mean gradient contribution in (2.13), the variance of $\mathrm{D}\phi/\mathrm{D}t$ in the theory is somewhat of an underestimate, which in turn leads to (2.15) being an underestimate for $\tau_{L,\phi}/\tau_\eta$ at low or moderate Reynolds number.

The results presented above indicate that the hypothesis of small scales being

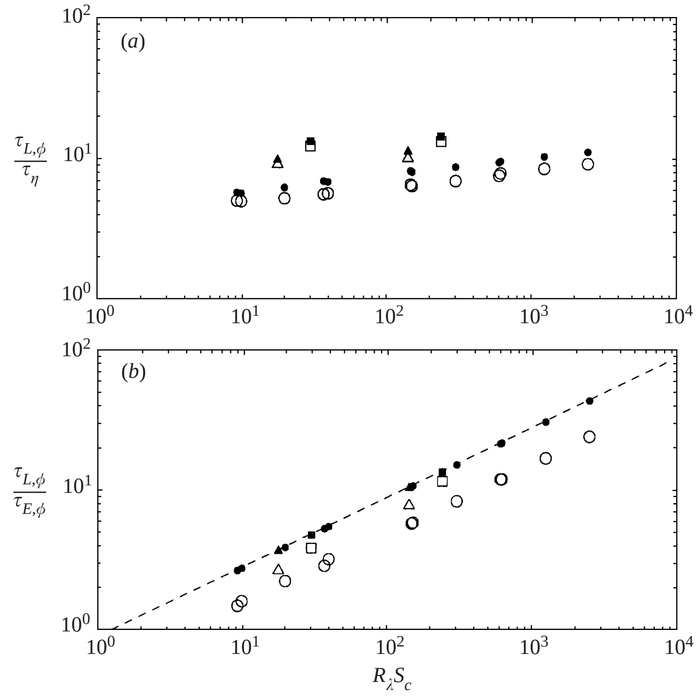

FIGURE 2. Same as figure 1, but for (*a*) normalized Lagrangian scalar Taylor time scale ($\tau_{L,\phi}/\tau_\eta$) and (*b*) the ratio $\tau_{L,\phi}/\tau_{E,\phi}$. Theoretical estimates (closed symbols for $\tau_{L,\phi}/\tau_{E,\phi}$) are based on (2.15) and (2.16). (The latter is also given by the dashed line in (*b*).)

advected nearly unchanged by the large scales (Tennekes 1975) can be extended to passive scalars. Use of Lagrangian methods is supported by both the smaller rate of change variance and longer time scales in this reference frame. In contrast Eulerian calculations involve two large terms (including the advective term which requires additional closure assumptions) which nearly cancel each other. It is interesting to note that, in the context of (2.2), the correlation coefficient between unsteady and advective terms for the scalar field is closer to -1.0 than those for the velocity field (-0.699 and -0.909 at R_λ 38 and 240 respectively, given in Tsinober *et al.* 2001). In other words, the hypothesis considered here appears to be more valid for the scalars than for the velocity. Implications for the functional forms of Eulerian and Lagrangian spectra are considered in the next section.

3. Eulerian and Lagrangian spectra

If scalar fluctuations are assumed to be transported purely by being passively swept past a stationary observer at a constant speed, changes in space and time would be directly related via a simple transformation of independent variables through a typical advection velocity. In isotropic turbulence we expect this advective speed, say u^*, to be proportional, but not necessarily equal, to the r.m.s velocity (u'). One may suggest a simple correspondence between wavenumber (k) and frequency (ω) of the form

$$k \sim \omega/u^*. \tag{3.1}$$

This consideration also suggests that the Eulerian wavenumber spectrum ($E_\phi(k)$) and Eulerian frequency spectrum ($E^E_\phi(\omega)$) should have the same shape and correspond to

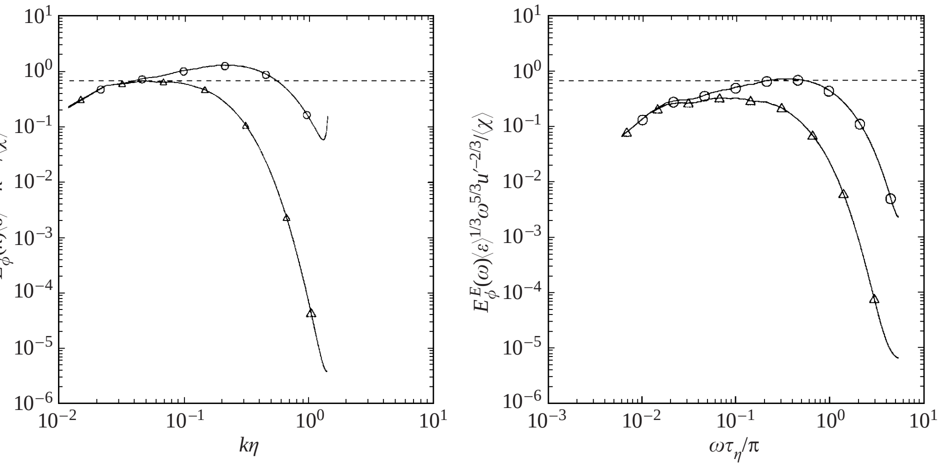

FIGURE 3. Normalized Eulerian wavenumber (a) and frequency (b) spectra of passive scalars at R_λ 140: $\triangle$, $Sc = 1/8$; $\bigcirc$, $Sc = 1$. A dashed line is drawn at 0.67 for reference.

each other as

$$E_\phi^E(\omega) \sim E_\phi(k)/u^*. \tag{3.2}$$

For $Sc \leqslant 1$ and sufficiently high Reynolds number it is well known that an inertial–convective range of wavenumbers should exist, such that

$$E_\phi(k) = \beta_E \langle\chi\rangle\langle\varepsilon\rangle^{-1/3}k^{-5/3} \quad \text{for} \quad 1/L \ll k \ll 1/\eta_{OC}, \tag{3.3}$$

where L is an integral length scale of the flow, $\eta_{OC} \equiv \eta Sc^{-3/4}$ is the Obukhov–Corrsin scale representing the smallest scale present in the scalar field, and β_E is the Obukhov–Corrsin constant. It follows from the relations above that the Eulerian frequency spectrum in the corresponding scaling range can be written as

$$E_\phi^E(\omega) = \beta_E(u^*/u')^{2/3}\langle\chi\rangle\langle\varepsilon\rangle^{-1/3}u'^{2/3}\omega^{-5/3}. \tag{3.4}$$

It should be noted, however, that the presence of u^* indicates a direct effect of parameters of the large scales, and implies that $E_\phi^E(\omega)$ does not satisfy inertial scaling, nor the associated ideas of small-scale universality.

Eulerian frequency spectra can be obtained in DNS by applying standard time-series analysis to flow variables at a subset of grid points saved over an extended period of time and at intervals small compared to the Kolmogorov time scale. Because this type of data was not saved in our 512^3 simulations at R_λ 240, we make our comparisons between $E_\phi(k)$ and $E_\phi^E(\omega)$ here at R_λ 140, the highest Reynolds number for which both types of data have been retained. Compensated spectra for both $E_\phi(k)$ and $E_\phi^E(\omega)$ in dimensionless forms according to the suggested scalings in (3.3) and (3.4) are shown in figure 3. A strong similarity in shape as predicted by the random-sweeping hypothesis is clearly evident.

The form of $E_\phi(k)$ itself in figure 3(a) deserves a few more comments. The data are characterized by a narrow scaling range (where the compensated spectrum is nearly flat) developing at around $k\eta \approx 0.03$, as well as a spectral 'bump' for $Sc = 1.0$ with its peak at $k\eta \approx 0.2$. In DNS $E_\phi(k)$ is computed as a three-dimensional spectral density, where k is the radius of a spherical shell in Fourier space, whereas in experiments usually a one-dimensional spectrum is measured. Relations between one- and three-dimensional spectra of scalar fields (Monin & Yaglom 1975) imply that the Obukhov–Corrsin scaling constant in the three-dimensional spectrum should be equal to 5/3 of that in the one-dimensional case. Accordingly, a dashed line at level $(0.4)(5/3) = 0.67$ corresponding to the data surveyed by Sreenivasan (1996) is drawn for comparison. The agreement observed is quite satisfactory, especially if allowance is made for the fact that (e.g. Yeung & Zhou 1997) isotropy relations are not perfectly satisfied at the relatively low wavenumbers where the present scaling range occurs.

The spectral bump seen in figure 3 has a physical origin similar to that caused by viscous effects in the velocity field (Falkovich 1994), but apparently occurs at a higher normalized wavenumber than in grid turbulence experiments (Mydlarski & Warhaft 1998). However, additional checks in DNS show that the bump inherently occurs at higher wavenumbers in the three-dimensional spectrum compared to its one-dimensional counterpart. A similar shift occurs with increasing Schmidt number, which at 1.0 in figure 3 is higher than that for temperature fluctuations with Prandtl number 0.7 in wind-tunnel experiments. Consequently, given the Reynolds number limitations in DNS, our results on the scalar spectrum can be said to be in broad agreement with experiment.

A rough assessment of the ratio u^*/u' appearing in (3.4) can be made by comparing the different scaling levels seen in the two parts of figure 3. The observed scaling level for $E_\phi^E(\omega)$ is approximately half of that seen in $E_\phi(k)$, which can be interpreted as suggesting $u^*/u' \approx (0.5)^{3/2} = 0.35$. This is in contrast with an advective speed of $3.1u'$ in a theoretical model for turbulence which is periodic in both space and time (L'vov, Pomyalov & Procaccia 1999). However, it is worth noting that the advective speed for velocity fluctuations inferred from DNS results for the Eulerian energy frequency spectrum (which is omitted here for lack of space) is also less than the r.m.s. velocity and of the same order as given here for scalar fluctuations.

The advection hypothesis of Tennekes (1975) implies that the highest Eulerian frequency for the velocity fluctuations is of the order $u'/\eta \sim R_\lambda^{1/2}/\tau_\eta$, compared to $1/\tau_\eta$ for the Lagrangian frequency. For the scalars the highest Eulerian frequency can be replaced by $u'/\eta_{OC} \sim Sc^{3/4}R_\lambda^{1/2}/\tau_\eta$. As a result, and also according to the time-scale ratio estimate in (2.16), the Eulerian frequency spectrum is expected to have more high-frequency content than the corresponding Lagrangian spectrum, especially for scalars of higher Schmidt number. In figure 4 we compare these spectra, both for passive scalars ($E_\phi^E(\omega)$ and $E_\phi^L(\omega)$, panel *a*) and the velocity ($E^E(\omega)$ and $E^L(\omega)$, panel *b*). At high frequencies the Lagrangian spectra are subject to some numerical noise, which may be a result of interpolation errors involved in obtaining Lagrangian values in numerical simulations of finite resolution. Nevertheless, it is clear that whereas Eulerian and Lagrangian spectra are comparable in magnitude at low frequencies, the fall-off at high frequencies is substantially slower in the Eulerian spectrum. The contrast is stronger for the scalars than for the velocity: e.g. for $Sc = 1$ at $\omega\tau_\eta/\pi = 1$, $E_\phi^E(\omega)$ is about 2.5 orders of magnitude higher than $E_\phi^L(\omega)$, compared to a factor of less than 20 between $E^E(\omega)$ and $E^L(\omega)$. Also, as we argued above, the Eulerian spectrum for $Sc = 1$ extends to even higher frequencies than for $Sc = 1/8$. The comparison here thus demonstrates a characteristic spectral broadening in Eulerian frequency spectra due to advective effects.

The scaling behaviour of the Lagrangian frequency spectrum of the scalar is, on the other hand, less well understood. Arguments based on dimensional analysis in the classical inertial range suggest $E_\phi^L(\omega) \sim \langle\chi\rangle\omega^{-2}$. However, the available data (Yeung 2001) have been inconclusive, with no clear evidence of an asymptotic scaling range. Similar uncertainties, although to a lesser degree, are also encountered in results on

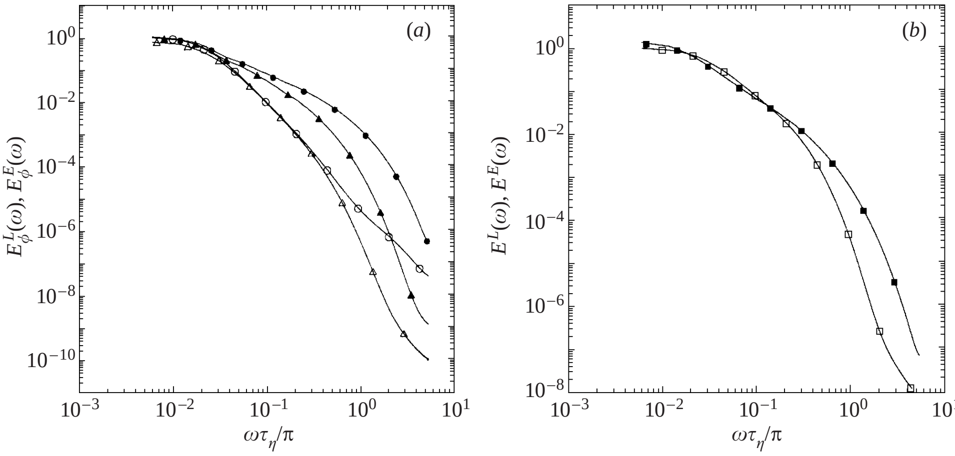

FIGURE 4. Comparisons, in unnormalized form, of Lagrangian (open symbols) and Eulerian (closed symbols) frequency spectra of (*a*) passive scalars and (*b*) the velocity, at R_λ 140: △, $Sc = 1/8$; ○, $Sc = 1$; □, for the velocity.

the Lagrangian frequency spectrum of the velocity. These difficulties are a reflection of the fact that the range of time scales in turbulence increases more slowly with the Reynolds number than the range of length scales, which has as a general consequence that (see e.g. Yeung 2002) inertial scaling in Lagrangian statistics generally requires higher Reynolds numbers than for Eulerian quantities.

4. Conclusions

Our main theme in this paper is to examine the applicability and implications of random-sweeping concepts based on the advection hypothesis of Tennekes (1975) when extended to the transport of passive scalars in turbulent flow. Data from direct numerical simulations have indicated strong mutual cancellation between local and advective derivatives consistent with the scenario of small scales being passively swept along by the large-scale motions. Theoretical estimates are developed (equations (2.6), (2.15) and (2.16)) and are compared with DNS data on the time scales for Eulerian and Lagrangian rates of change (Yeung 2001) including their Reynolds and Schmidt number dependence. Correct trends are verified using DNS data at various Reynolds and Schmidt numbers with a uniform mean scalar gradient. Better agreement is consistently observed at higher Reynolds number, where all of the assumptions made, including de-coupling of the scalar field microstructure from the large-scale velocity, local isotropy at the second-moment level, and the scaling of molecular diffusion, are expected to be more accurate.

In this paper we have investigated the consequences of the random sweeping hypothesis on the relationships among three different versions of the scalar spectrum, namely the Eulerian wavenumber spectrum $[E_\phi(k)]$, the Eulerian frequency spectrum $[E_\phi^E(\omega)]$, and the Lagrangian frequency spectrum $[E_\phi^L(\omega)]$. We find that, in accordance with expectations based on the hypothesis, the Eulerian frequency spectrum of passive scalars is (figure 3) very similar in form to the Eulerian wavenumber spectrum. On the other hand, the Eulerian frequency spectrum differs substantially (figure 4) from the Lagrangian frequency spectrum, especially in a spectral broadening towards higher frequencies for scalars of higher Schmidt number. However, the scaling of the

Lagrangian spectrum is not yet understood, and will require data at higher Reynolds numbers than currently available.

Overall, our theoretical and numerical results confirm that the random-sweeping hypothesis is valid for passive scalars, perhaps even more so than for the velocity field. We emphasize that the validity of the hypothesis lends support to the use of Lagrangian approaches to describe turbulent transport processes.

P. K. Y. gratefully acknowledges support from the National Science Foundation, via Grant CTS-9705678, and via NSF cooperative agreement ACI-9619020 through computing resources provided by the National Partnership for Advanced Computational Infrastructure at the San Diego Supercomputer Center. Valuable comments by the referees are also highly appreciated.

REFERENCES

Belinicher, V. I. & L'vov, V. S. 1987 A scale-invariant theory of fully developed hydrodynamic turbulence. *Sov. Phys. JETP* **66**, 303–313.

Fairhall, A. L., Gat, O., L'vov, V. & Procaccia, I. 1996 Anomalous scaling in a model of passive scalar advection: exact results. *Phys. Rev.* E **53**, 3518–3535.

Falkovich, G. 1994 Bottleneck phenomenon in developed turbulence. *Phys. Fluids* **6**, 1411–1414.

Kerr, R. M. 1985 Higher-order derivative correlations and the alignment of small scale structures in isotropic numerical turbulence. *J. Fluid Mech.* **153**, 31–58.

Kraichnan, R. H. 1966 Isotropic turbulence and inertial range structure. *Phys. Fluids* **9**, 1728–1752.

L'vov, V. S., Pomyalov, A. & Procaccia, I. 1999 Temporal surrogates of spatial turbulent statistics: The Taylor hypothesis revisited. *Phys. Rev.* E **60**, 4175–4184.

Monin, A. S. & Yaglom, A. M. 1975 *Statistical Fluid Mechanics*, Vol. II (ed. J. L. Lumley). MIT Press.

Mydlarski, L. & Warhaft, Z. 1998 Passive scalar statistics in high-Péclet-number grid turbulence. *J. Fluid Mech.* **358**, 135–175.

Overholt, M. R. & Pope, S. B. 1996 Direct numerical simulation of a passive scalar with imposed mean gradient in isotropic turbulence. *Phys. Fluids* **8**, 3128–3148.

Sawford, B. L. 2001 Turbulent relative dispersion. *Annu. Rev. Fluid Mech.* **33**, 289–317.

Sreenivasan, K. R. 1996 The passive scalar spectrum and the Obukhov–Corrsin constant. *Phys. Fluids* **8**, 186–196.

Tennekes, H. 1975 Eulerian and Lagrangian time microscales in isotropic turbulence. *J. Fluid Mech.* **67**, 561–567.

Tsinober, A., Vedula, P. & Yeung, P. K. 2001 Random Taylor hypothesis and the behavior of local and convective accelerations in isotropic turbulence. *Phys. Fluids* **13**, 1974–1984.

Vedula, P. Yeung, P. K. & Fox, R. O. 2001 Dynamics of scalar dissipation in isotropic turbulence: a numerical and modeling study. *J. Fluid Mech.* **433**, 29–60.

Yeung, P. K. 2001 Lagrangian characteristics of turbulence and scalar transport in direct numerical simulations. *J. Fluid Mech.* **427**, 241–274.

Yeung, P. K. 2002 Lagrangian investigations of turbulence. *Annu. Rev. Fluid Mech.* **34**, 115–142.

Yeung, P. K. & Pope, S. B. 1989 Lagrangian statistics from direct numerical simulations of turbulence. *J. Fluid Mech.* **207**, 531–586.

Yeung, P. K. & Zhou, Y. 1997 On the universality of the Kolmogorov constant in numerical simulations of turbulence. *Phys. Rev.* E **56**, 1746–1752.